Code No.2882/AICTE

FACULTY OF ENGINEERING

B. E. (AICTE) II – Semester (Suppl.) Examination, December 2019

Subject: Mathematics - II

Time: 3 hours **Max. Marks: 70**

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (20 Marks)

1. Reduce the following matrix to row echelon form and find its rank. 2

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 2 \\ 5 & -5 & 11 \end{bmatrix}$$

2. Find the spectral radius of the matrix. 2

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

3. Under what condition the following differential equation is exact. 2

$[f(x) + g(y)]\, dx + [h(x) + d(y)]\, dy = 0$

4. Write Riccati's equation and Clairaut's equation. 2

5. Find the orthogonal trajectories of the family of curve $x^2+16y^2 = c$. 2

6. Find the solution of the initial value problem. 2

$4y'' - 8y' + 3y = 0; \quad y(0) = 1, \quad y'(0) = 3$

7. It is known that $\frac{1}{x}$ is a solution of the differential equation $x^2y'' + 4xy' + 2y = 0$. Find the second linearly independent solution and write the general solution. 2

8. Using Beta and Gamma functions, evaluate the integral 2

$\int_{-1}^{1}(1-x^2)^n dx$ where 'n' is a positive integer.

9. Express $3P_3(x) + 2P_2(x) + 4P_1(x) + 5P_0(x)$ as a polynomial in x, where $P_m(x)$ is the Legendre Polynomial of order 'm'. 2

10. Solve the following initial value problem using Laplace transform 2

$y'' + 4y = 0; \quad y(0) = 1, \quad y'(0) = 6.$

PART – B (50 Marks)

11. Find the eigenvalues and the corresponding eigenvectors of the given matrix. 10

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

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12. (a) Define the integrating factor of the non-homogeneous first order linear equation 1

$\frac{dy}{dx} + p(x)y = r(x)$.

(b) The initial value problem governing the current 'i' flowing in a series RL circuit when a voltage v(t) = t is applied, is given by 4

$iR + L\frac{di}{dt} = t, \quad t \geq 0; \quad i(0) = 0$

where R and L are constants. Find the current i(t) at time t.

(c) Find the general solution of the equation $\frac{dy}{dx} = 2xy^2 + (1-4x)y + 2x - 1$, if y=1, is a solution. 5

13. (a) Show that e^x, e^{2x}, e^{3x} are the fundamental solutions of 5

$y''' - 6y'' + 11y' - 6y = 0$ on any interval I.

(b) Find the general solution of the equation 5

$y'' + y = \sec x$ by the method of variation of parameters.

14. (a) Show that the following recurrence relation. 8

$(n+1)\, P_{n+1}(x) = (2n+1)x\, P_n(x) - n\, P_{n-1}(x)$.

Using above relation obtain Legendre Polynomials $P_2(x)$; $P_3(x)$; $P_4(x)$ given that $P_0(x)=1$ and $P_1(x) = x$.

(b) Show that $P_2(x)$ and $P_3(x)$ are orthogonal functions on [-1, 1]. 2

15. (a) Find the Laplace transformation of the Piecewise continuous function 5

$$f(t) = \begin{cases} 0; & 0 \leq t \leq 2 \\ k; & t \geq 2; \text{ k is a constant} \end{cases}$$

(b) Find the inverse Laplace transform of 5

$$\frac{5s+3}{(s-1)(s^2+2s+5)}.$$

16. Reduce the quadratic form Q = 2(xy + yz + zx) to canonical form by orthogonal transformation and find its nature. 10

17. (a) Find the orthogonal trajectories of the family of the Confocal Conics $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$ where λ is the parameter. 5

(b) Find the orthogonal trajectories of the cardioids $r = a(1 - \cos\theta)$. 5

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Code No. 2882/AICTE/M

FACULTY OF ENGINEERING

B.E. II-Semester (AICTE) (Main & Backlog) Examination, November 2020

Subject : Mathematics - II

Time : 2 Hours **Max. Marks: 70**

Note: Answer Any five Questions from Part-A & Any Four Questions From Part-B.

PART – A (5x4=20 Marks)

1 Examine whether the vector (1, 2,), (3, 4), (3, 7) are linearly independent.

2 If 1, -1, 2 are the eigen values of a 3 x 3 matrix A, find the determinant of the matrix $A^3 - 2A^{-1} + I$.

3 Define exact differential equation.

4 Find the singular solution of the Clairant's equation $y + xy' - \frac{1}{y'}$.

5 Find the complementary function of $(D^2 + D + 1)^2 y = e^x \tan x$.

6 Solve $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

7 Evaluate $\Gamma\left(-\frac{3}{2}\right)$.

8 State Rodrigue's formula and hence find $P_2(x)$.

9 Find $L\{e^{-t} \sin t \cos t\}$

10 Evaluate $\int_0^\infty \frac{\sin t}{t}\,dt$ using Laplace transform.

PART – B (4x15=60 Marks)

11 (a) Test for consistency and hence solve the following system of equations.

$x_1 + 2x_2 + x_3 = 2,\quad 3x_1 + x_2 - 2x_3 = 1,\quad 4x_1 - 3x_2 - x_3 = 3,\quad 2x_1 + 4x_2 + 2x_3 = 4$

(b) Find the characteristics equation of $A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{pmatrix}$ and hence find A^{-1}.

12 (a) Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ .p

(b) Find the orthogonal trajectories of the family of parabolas $y^2 = 2cx + c^2$.

13 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - y = (e^x + 1)^2.$$

(b) Solve $y'' + 2y' + 2y = e^{-x}\cos x$ by the method of variation of parameters.

14 (a) Evaluate $\int_0^1 \frac{dx}{\sqrt{1 - x^4}}$ using Beta and Gamma functions.

(b) Show that $P_{2n}(0) = (-1)^n \frac{1.3.5......(2n-1)}{2.4.6.......2n}$ and $P_{2n+1}(0) = 0$.

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15 (a) Find the inverse Laplace transform of $\log\left(\frac{s + a}{s + b}\right)$.

(b) Apply Laplace transforms to solve $y'' + y = 3\cos 2x$, $y'(0) = 0 = y(0)$.

16 Reduce the quadratic form $Q = 2(xy + yz + zx)$ to Canonical form using orthogonal transformation.

17 (a) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(b) Apply convolution theorem to find $L^{-1}\left\{\frac{s}{(s^2 + 1)(s - 1)}\right\}$.

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Code No. 15265/AICTE

FACULTY OF ENGINEERING

B.E. II - Semester (AICTE) (Main) Examination, October 2021

Subject: Mathematics - II

Time: 2 Hours

Max. Marks: 70

Note: i) First Question is compulsory and answer any three questions from the remaining six questions.

ii) Answers to each question must be written at one place only and in the same order as they occur in the question paper.

iii) Missing data, if any, may suitably be assumed.

Answer any four questions from the following. **(4x4=16 Marks)**

1 a Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 1 & 3 & -1 \\ 8 & 13 & 14 \end{bmatrix}$.

b Solve $y(2xy + e^x)dx = e^x dy$.

c Solve $(D^2 + 9)y = \sin 3x$.

d Evaluate $\int_0^\infty e^{-3x}(1-e^{-x})^5 dx$ in terms of beta function.

e Find $L\{t^3 e^t + \sin^2 t\}$.

f Find $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+3)}\right\}$.

g Evaluate $6P_3(x) + 4P_2(x) - 16P_1(x)$ as a polynomial of x.

(3x18=54 Marks)

2 (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.

(b) Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 + 12x_1x_2 + 4x_1x_3 - 8x_2x_3$ into canonical form.

3 (a) Solve $y(x+y)dx - x^2dy = 0$.

(b) Solve $y(2xy+1)dx + x(1+2xy-x^3y^3)dy = 0$.

4 (a) Solve $y'' + 4y = x\cos x$.

(b) Solve $y'' + 2y' + y = e^{-x}\log x$ by the method of variation of parameters.

5 (a) Find the power series solution of the differential equation $y'' + 2xy' + y = 0$ about the origin.

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(b) Evaluate $\frac{d}{dx}\left[erf(\alpha x)\right]$.

6 (a) Find $L\left\{\int_0^t ue^{-u}\sin 4u\,du\right\}$.

(b) Find $L^{-1}\left\{\frac{1}{s^3(s+2)}\right\}$.

7 (a) Find the orthogonal trajectories of the family of curves $y^3+3x^2y=c$ where c i arbitrary constant.

(b) Solve $x^2y''-xy'-3y=x^2\log x$.

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Code No. 15010/AICTE

FACULTY OF ENGINEERING

B.E. (AICTE) II-Semester (Backlog) Examination, July 2021

Subject : Mathematics – II

Time: 2 hours **Max. Marks: 70**

Note: Missing data, if any, may be suitably assumed.

PART – A

Answer any five questions. **(5x2 = 10 Marks)**

1 Define rank of a matrix. Find the rank of the matrix

$$A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 2 & -4 \end{pmatrix}$$

2 Determine the nature of the quadratic form $Q = 2(x^2 + xy + y^2)$.

3 Solve $y' + y\tan x = \cos x$.

4 Find the orthogonal trajectories of the family of curves $r = a\theta^2$.

5 Obtain a particular integral of $y'' + y = \sin^2 x$.

6 If $y_1 = x^4$ is a solution of $x^2y'' - 2xy' - 4y = 0$, find the second linearly independent solution.

7 Find the value of $P_n'(-1)$.

8 Show that erf(x) + erfc(x) = 1.

9 Find the Laplace transform of $f(t) = e^{-t}\cosh 2t$.

10 If $L\{f(t)\} = \cot^{-1} s$, find $f(t)$.

PART – B

Answer any four questions. **(4x15 = 60 Marks)**

11 (a) Determine whether the vectors (2, 3, 1, -1), (2, 3, 1, 2), (4, 6, 2, 1) are linearly dependent.

(b) If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, then find the matrix A^{32} using Cayley-Hamilton theorem.

12 (a) Solve $(y - 2x^3)dx - x(1 - xy)dy = 0$.

(b) Solve $\frac{dy}{dx} = 4x^3(y - x)^2 + \frac{y}{x}$ if $y = x$ is a solution.

13 (a) Solve $y'' - 4y' + 4y = 8x^2e^{2x}\sin 2x$.

(b) Find the general solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2)$.

14 (a) Show that $\beta(m,n) = \frac{\Gamma m \; \Gamma n}{\Gamma(m+n)}$.

(b) Find the power series solution of the differential equation $y'' + y = 0$ about $x = 0$.

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15 (a) Find the Laplace transform of $f(t)=\dfrac{e^{2t}-e^{3t}}{t}$.

(b) Using convolution theorem, find $L^{-1}\left\{\dfrac{1}{s^2(s-4)}\right\}$.

16 (a) Find the eigen values and the corresponding eigen vectors of the matrix

$$A=\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}$$

(b) Solve $y''-y=e^{t}$ by the method of variation of parameters.

17 Using Laplace transform, solve $y''+2y'+5y=e^{-t}\sin t,\ y(0)=0,\ y'(0)=1$.

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Code No. 15010/AICTE/BL

FACULTY OF ENGINEERING

B.E. II - Semester (AICTE) (Backlog) Examination, October 2021

Subject: Mathematics - II

Time: 2 Hours **Max. Marks: 70**

(Missing data, if any, may be suitably assumed)

PART – A

Note: Answer any five questions. **(5x2 = 10 Marks)**

1 Obtain the symmetric matrix for the quadratic form $2x_1^2 + 3x_1x_2 + x_2^2$.

2 Show that sum of eigen values of a matrix is its trace and product of eigen values of a matrix is its determinant.

3 Find the integrating factor of the differential equation
$x\frac{dy}{dx} = 2y + x^4 + 6x^2 + 2x,\ x \neq 0$.

4 Define orthogonal trajectory of a given family of curve and write the procedure to find it in polar coordinates.

5 Explain method of variation of parameters.

6 Solve $(D^3 - a^3)y = 0$, where $D = \frac{d}{dx}$.

7 Define Gamma and Beta functions.

8 Express $f(x) = 6x^2 - 5x + 3$ in terms of Legendre polynomials.

9 Find the inverse Laplace transform of the function $\frac{s}{(s+4)^3}$.

10 Find the Laplace transform of the function $f(t) = t^2 e^{3t}$.

PART – B

Note: Answer any four questions. **(4x15 = 60 Marks)**

11 (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and find A^{-1}, if it exists.

(b) Find rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$.

12 (a) Find the general solution of the differential equation
$y' = y^2 - (2x-1)y + x^2 - x + 1$ if $y = x$ is a solution of the differential equation.

(b) Find the equation of the family of all orthogonal trajectories of the family of circles which pass through the points (2,0), (-2,0).

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13 (a) Find the general solution of the equation $y'' + 16y = 32\sec 2x$. using the method of variation of parameters.

(b) Find the general solution of the differential equation $x^2y'' + 4xy' + 2y = 0(x > 0)$.

14 (a) Show that $\beta(m,n) = \frac{\gamma(m)\gamma(n)}{\gamma(m+n)}$.

(b) Express $\int_0^1 x^{3/2}\left(1-\sqrt{x}\right)^{1/2} dx$ is terms of Beta function.

15 (a) Use the Laplace transforms to solve the initial value problem $12y'' - 24y' + 9y = 2t, y(0) = 0, y'(0) = 3$.

(b) Find the inverse Laplace transform of the function $\frac{s}{s^2 + 4s + 8}$.

16 (a) Find the general solution of the equation $y'' + 6y = 6\cos x$.

(b) Reduce the quadratic form $2xy + 2yz + 2zx$ into canonical form.

17 Solve $(D^3 + 3D^2 - 4D - 12)y = e^{2x} + \sin 3x + 4\cos 2x$.

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Code No. D-2333/N/AICTE

FACULTY OF ENGINEERING

B.E. II - Semester (AICTE) (BACKLOG) Examination, March / April 2022

Subject: Mathematics - II
(Common for All Branches)

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each question carries 14 marks.
(ii) Answer to each question must be written in one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1 (a) Find the rank of the matrix $A=\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\\0 & 2 & 1\end{pmatrix}$.

(b) Find the values of a and b so that the differential equation $(x^2+axy-2y^2)dx+(y^2-4xy+bx^2)dy=0$ is exact.

(c) Find a differential equation of the form $ay''+by'+cy=0$ for which the functions $1, e^{-x}$ are solutions.

(d) Define error and complementary functions.

(e) Find $L^{-1}\left\{\frac{1}{s^2+2s+2}\right\}$.

(f) State Cayley-Hamilton theorem.

(g) Solve $2x^2y''+xy'-6y=0$.

2 (a) Determine whether the vectors (1,1,0,1), (1,1,1,1), (4,4,1,1,), (1,0,0,1) are linearly dependent.

(b) Reduce the quadratic form $Q=3x^2-2y^2-z^2-4xy+12yz+8xz$ into canonical form and find the nature of the quadratic form.

3 (a) Solve $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$.

(b) Find the general solution of the Riccati's equation $y'=4xy^2+(1-8x)y+4x-1$, if $y=1$ is a particular solution.

4 (a) Solve $y''+2y'+2y=x^3+6\cos^2 x$.

(b) Apply the method of variation of parameters to solve $y''+y=\tan x$.

5 (a) Evaluate the following integrals using Beta and Gamma functions.

(i) $\int_0^\infty 2^{-4x^2}dx$ (ii) $\int_{-1}^{1}(1-x^2)^n dx$, where n is a positive integer.

(b) Using Rodrigue's formula, find $P_0(x), P_1(x), P_2(x), P_3(x)$ and hence express $6P_0(x)-7P_1(x)+8P_2(x)+3P_3(x)$ as a polynomial in x.

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Code No. D-2333/N/AICTE

6 (a) Find the Laplace transform of the following functions.

(i) $\frac{\sinh t}{t}$ (ii) $te^{-t}\cos t$.

(b) Apply Laplace transforms to solve $y''+3y'+2y=e^{-t}$, $y(0)=0$, $y'(0)=-1$.

7 (a) Find the characteristic equation of the matrix $A=\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and hence find A^4.

(b) Find the series solution of $(1-x^2)y''-2xy'+6y=0$ about $x=0$.

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Code No. D-2010/O/AICTE

FACULTY OF ENGINEERING

B.E. II - Semester (AICTE) (Backlog) Examination, March / April 2022

Subject: Mathematics – II
(Common for All Branches)

Time: 3 Hours **Max. Marks: 70**

(Missing data, if any, may be suitably assumed)

PART – A

Note: Answer all questions. **(10 x 2 = 20 Marks)**

1. Define rank of the matrix and find the rank of the matrix A= $\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$
2. Let $V_1 = (1,-1,0)$ $V_2 = (0,1,-1)$ and $V_3 = (0,0,1)$ be elements of R^3. Show that the set of vectors $\{V_1, V_2, V_3\}$ is linearly independent.
3. Define exact differential equation.
4. Obtain the singular solution of the equation $y = xy' + (y')^2$.
5. Find a differential equation of the form $ay'' + by' + cy = 0$, for which the function $1, e^{-2x}$ are solutions.
6. Define Cauchy- Euler equation.
7. Express the sums of Legendre polynomial $8P_4(x) + 2P_2(x) + P_0(x)$ in terms of powers of x.
8. Show that $\Gamma(-1/2) = -2\sqrt{\pi}$
9. Find the Laplace transform of the function $t\ sin4t$
10. Define convolution theorem in Laplace transform.

PART – B

Note: Answer any five questions. **(5 x 10 = 50 Marks)**

11. (a) Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$, have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.
    (b) Reduce the quadratic form $2xy + 2yz + 2zx$ into canonical form.
12. (a) Solve the initial value problem $e^x(cosy\ dx - siny\ dy) = 0,\ y(0) = 0$.
    (b) Solve the differential equation $\frac{dy}{dx} - y = y^2(sinx + cosx)$.
13. (a) Solve $(D^2 + 3D + 2)y = xe^x\ sinx$.
    (b) It is known that e^{-2x} is a solution of the differential equation $y'' - y' - 6y = 0$. Find the second linearly independent solution and write the general solution.

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14. (a) Prove that $(n+1)\ P_{n+1}(x) = (2n+1)\,x\ P_n(x) - nP_{(n-1)}(x)$.

(b) Evaluate $\int_0^\infty \sqrt{x}\ e^{-x^2}\,dx$.

15. (a) Use the Laplace transforms to solve the initial value problem

$y'' + 2y' + 5y = 1 + t, y(0) = 4, y'(0) = -3$.

(b) Find the inverse Laplace transform of the function $\frac{5s+6}{(s-1)^2}$.

16. (a) Find the general solution of the equation $y'' - 2y' - 3y = 3e^{2x}$.

(b) Find the eigen values and the corresponding eigen vectors of the Matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

17. Solve $(D^2 - 2D - 3)y = x + e^{2x}\cos 2y + \sin 5x$.

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Code No: E-5612/N/AICTE

FACULTY OF ENGINEERING

B.E. II-Semester (AICTE) (Backlog) (New) Examination, February / March 2023

Subject: Mathematics-II

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each questions carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. (a) Determine whether the vectors $(1,-1,2),(2,3,0),(4,1,4)$ are linearly dependent.
   (b) Find the singular solution of the Clairaut's equation $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$.
   (c) Solve $x^2y'' - 2xy' + 2y = 0$.
   (d) Prove that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta \; d\theta$.
   (e) Find $L\left\{\frac{\sin ht}{t}\right\}$.
   (f) Find the rank of matrix $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \end{bmatrix}$.
   (g) Find the orthogonal trajectories of the family of curves $x^2 - y^2 = c$, where c is a parameter.

2. Reduce the quadratic form $Q = 3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical form and find its nature.

3. (a) Solve $(3xy - 2y^2)dx + (x^2 - 2xy)dy = 0$.
   (b) Solve $(x + 2y^3)\frac{dy}{dx} = y$.

4. (a) Find the general solution of $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = e^{2x}(1 + x)$.
   (b) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + y = x$.

5. (a) Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$.
   (b) Find the series solution of $y' - 4y = 0$ about $x = 0$.

6. (a) Find $L\{t \sin 2t \; \cos 4t\}$.
   (b) Using Laplace transform, solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = 0$, $y(0) = 0$, $y'(0) = 4$.

7. (a) Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and hence find A^{-1}.
   (b) Express $3x^3 + x^2 - 2x + 4$ in terms of Legendre polynomials.

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Code No. E-5612/N/BL/AICTE

FACULTY OF ENGINEERING

B.E. (Common to all Branches) II Semester (AICTE) (Main & Backlog) Examination, September / October 2023

Subject: Mathematics-II

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each questions carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. (a) If the sum of the eigen values of $A=\begin{bmatrix} 5 & 7 & 3 \\ -2 & k & 5 \\ 0 & 3 & 2 \end{bmatrix}$ is -10, then find k.

(b) Define an exact differential equation.

(c) Solve $x^2y'' - 2xy' - 4y = 0$.

(d) Evaluate $\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta$ using Gamma and Beta functions.

(e) Find $L\{e^{-4t}\, t^2\}$.

(f) Find the matrix of the quadratic form

$$Q = 2x_1^2 + 4x_2^2 + 5x_3^2 - 6x_1x_2 + 8x_2x_3 - 10x_3x_1.$$

(g) Obtain the singular solution of $y = xy' - \frac{1}{y'}$.

2. (a) Find the rank of the matrix $A=\begin{bmatrix} 2 & -1 & 3 & 2 \\ -4 & 0 & 3 & 5 \\ 7 & 2 & 1 & 1 \end{bmatrix}$ by reducing to echelon form.

(b) Verify Cayley-Hamilton theorem for $A=\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and hence find A^{-1}.

3. (a) Solve $(x^2 + y^3)dx - xy^2dy = 0$.

(b) Find the orthogonal trajectories of the family of circles passing through $(0,2)$ and $(0,-2)$.

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4. (a) Solve $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 3e^{-x} + 2x + \sin x$.

   (b) Solve $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation of parameters.

5. (a) Prove that $\beta(m,n) = \dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.

   (b) State Rodrigue's formula and hence find $P_0(x), P_1(x), P_2(x)$ and $P_3(x)$.

6. (a) Find (i) $L\left\{\frac{\sinh t}{t}\right\}$ and (ii) $L\{e^{-t}\sin^2 t\}$

   (b) Using Laplace transforms, solve $\frac{d^2y}{dt^2} + 25y = 10\cos 5t$, $y(0) = 2$, $y'(0) = 0$.

7. (a) Find the values of λ and μ for which the system equations $x + y + z = 3$, $x + 2y + 2z = 6$, $x + \lambda y + 3z = \mu$ has (i) no solution (ii) a unique solution and (iii) infinite number of solutions.

   (b) Using convolution theorem, find $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$.

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Code No: E – 5010/O/AICTE

FACULTY OF ENGINEERING

B.E. II - Semester (AICTE) (Backlog)(Old) Examination, February/ March 2023

Subject: Mathematics-II

Time: 3 Hours **Max. Marks: 70**

(Missing data, if any, may be suitably assumed)

PART – A

Note: Answer all the questions. **(10 x 2 = 20 Marks)**

1. Find the value of k such that rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2.
2. Show that the sum of eigen values of a matrix is its trace.
3. Is the Differential equation $(2xy + y - tany)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$ exact or not?
4. Solve $p = sin\,(y - xp)$, where $p = \frac{dy}{dx}$.
5. Solve $(D^3 + 1)y = 0$.
6. Define Legendre polynomial of first kind.
7. Write Cauchy Riemann equations in polar form.
8. State Cauchy's inequality and Liouville's theorem.
9. Find the nature and location of singularity of the function $\frac{z-\sin z}{z^2}$.
10. Evaluate $\int_{|z|=3} \frac{\sin z}{z^2 e^z}\, dz$.

PART – B

Note: Answer any five questions. **(5 x 10 = 50 Marks)**

11. Verify Cayley Hamilton Theorem for the Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ and hence find A^{-1}.
12. a) Solve $(1 + x^2)dx = (\tan^{-1} y - x)dy$.
    b) Solve $\frac{dy}{dx} + x\sin 2y = x^3 cos^2(y)$.
13. Solve $(D^2 - 4D + 4) = 8x^2 e^{2x} \sin 2x$.
14. a) Evaluate $\int_C \frac{\sin \pi z^2 \cos \pi z^2}{(z-1)(z-2)}\, dz$ using Cauchy's integral formula, where C is the circle $|z| = 3$.
    b) Evaluate $\int_C \frac{e^z}{(z^2+\pi^2)^2}\, dz$ where c is the circle $|z| = 4$.

..2..

15. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region
(a) $|z| < 4$ (b) $1 < |z| < 2$ (c) $|z| > 2$ (d) $0 < |z - 1| < 1$

16. Find the bilinear transformation which maps the points$(0,1,\infty)$ in the z -plane onto the points$(-1,-i,1)$ in the w-plane.

17. Find the orthogonal trajectory of the family of coaxial circles $x^2 + y^2 + 2\lambda x + c = 2$, λ being the parameter.

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Code No: F-13612/N/AICTE

FACULTY OF ENGINEERING

B.E. II - Semester (AICTE) (Backlog) (New) Examination, February/ March 2024

Subject: Mathematics-II

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each question carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. a) Find the symmetric matrix A for the quadratic form $x_1^2 + 2i x_1 x_2 - 8 x_1 x_3 + 4i\, x_2 x_3 + 4x_3^2$.
   b) Solve $cos^2 x \frac{dy}{dx} + y = tan x$
   c) Solve $(D^3 + 2D^2 + D)y = x^2 e^{2x} + sin^2 x$.
   d) Show that $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$ and find the value of $\Gamma(7/2)$.
   e) Express $6P_3(x) - 2P_1(x) + P_0(x)$ in terms of powers of x.
   f) Write Cayley -Hamilton theorem and verify it for the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.
   g) Find $L^{-1}\left\{\frac{s+3}{(s-1)(s+2)}\right\}$

2. a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & -4 \\ -1 & -3 & 4 \\ 2 & 6 & -5 \end{bmatrix}$ using elementary row operations.
   b) Find Eigen values and Eigen vectors for the matrix
   $A= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$.

3. a) Find the integrating factor and solve the differential equation
   $y(1 + xy^2)dx + 2(x^2y^2 + x + y^4)dy = 0,\ y(0) = 1.$
   b) Solve $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1 + x)^2$.

4. a) Solve $y'' - 2y' + y = e^x \log x$ by method of variation of parameters.
   b) Find the orthogonal trajectory of family of curves $r = c(1 + cos\,\theta)$.

5. a) Solve Legendre's differential equation $(1 - x^2)y'' - 2xy' + n(n + 1)y = 0, n \in Z^+$ and write its General solution.
   b) Write an expression for Legendre's polynomial $P_n(x)$, and find $P_0(x)$, $P_1(x)$ and $P_2(x)$.

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6. a) Test for consistency and solve the following system of equations:
   $2x - 3y + 7z = 5, 3x + y - 3z = 13, 2x + 19y - 47z = 32.$

   b) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_22x_3 + 4x_3x_1$ to canonical form through orthogonal transformation. Find index and signature

7. a) Write Convolution theorem, use it to solve the differential equation
   $y'' + 3y' + 2y = e^{-t}, y(0) = 0, y'(0) = -1$

   b) Apply Laplace transform to solve the initial value problem

$$y''' - 3y'' + 3y' - y = t^2e^t$$
$$y(0) = 1, y'(0) = 0, y''(0) = -2$$

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Code No: F-13010/O/BL/AICTE

FACULTY OF ENGINEERING

B.E. II Semester (AICTE) (Backlog) (Old) Examination, August/September 2024

Subject: Mathematics-II

Time: 3 Hours Max. Marks: 70

(Missing data, if any, may be suitably assumed)

PART – A

Note: Answer all the questions. (10 x 2 = 20 Marks)

1. Find the rank of the matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 7 & 22 \\ 2 & 6 & 8 \end{bmatrix}$ using elementary row operations.
2. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.
3. Find the integrating factor and hence solve the differential equation $(x^3y^2 + x)dy + (x^2y^3 - y)dx = 0$.
4. Find the general and singular solution of the Clauraut's equation $y' = px + (1 + p^2)$.
5. Find the general solution of the equation $y'' + 5y' + 6y = 2e^x$.
6. Solve $(D^4+18 D^2+81)y=0$.
7. Define Beta function and show that $\beta(m,n) = \beta(n,m)$.
8. Define Gamma Function and prove that $\Gamma(\alpha + 1) = \alpha\, \Gamma(\alpha)$
9. Show that $L\{f'''(t)\} = s^3F(s) - s^2 f(0) - sf'(0) - f''(0)$.
10. Find the $L\{e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t\}$.

PART – B

Note: Answer any five questions. (5 x 10 = 50 Marks)

11. Find Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Is it diagonizable, if so, write the modular matrix.

12. a) Use Gauss-Jordan method to find the inverse of the matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$.

    b) Test for consistency and solve the following system of equations:

    x+2y+z=3, 2x+3y+2z=5, 3x-5y+5z=2 and 3x+9y-z=4.

13. a) Find the orthogonal trajectory of the Cardioids r = a(1-cosθ).

    b) Solve y(logy)dx+(x-logy)dy=0.

..2..

14. a) $\frac{dy}{dx} + 4xy + xy^3 = 0$.

b) Solve $y'' - 2y' + y = e^x \log x$ by method of variation of parameters.

15. a) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = sin[2\,log(1+x)]$.

b) Find power series solution about the origin of the first order differential equation $y' = xy$.

16. Solve the differential equation $(1-x^2)y'' - 2xy' + 12y = 0$ using power series about the origin.

17. a) Solve the initial value problem $y''' - 5y'' + 7y' - 3y = 0,\ y(0) = 1,\ y'(0) = 0,\ y''(0) = -5$.

b) Write convolution theorem and solve the differential equation

$$y'' + 3y' + 2y = e^{-t}, y(0) = 0, y'(0) = -1.$$

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Code No: F-13612/N/BL/AICTE

FACULTY OF ENGINEERING

B.E. II Semester (AICTE) (Main & Backlog) (New) Examination, August/September 2024

Subject: Mathematics-II

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each question carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. a) Find the rank of the matrix Find rank of the matrix A= $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$.

   b) If $1,2,-1$are the eigen values of matrix A, Find trace of matrix $B = A - A^{-1} + A^2$.

   c) Define exact differential equation and solve $e^x(\cos y\, dx - \sin y\, dy) = 0$.

   d) Find the orthogonal trajectories of the family of curves $y = ce^x$, c is parameter.

   e) Solve $y'' + 2y' + 2y = 0$.

   f) Evaluate $L[e^{2t}\cos^2 t]$.

   g) Express the polynomial $3x^2 + 5x - 6$ in terms of Legendre polynomials.

2. a) Test for consistency and solve $4x - 3y - 9z + 6w = 0,\ 2x + 3y + 3z + 6w = 6,$ $4x - 21y - 39z - 6w = -24.$

   b) Verify Cayley- Hamilton theorem and find A^{-1} where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & -1 \\ -2 & -1 & 1 \end{bmatrix}$.

3. a) Solve $\frac{dy}{dx} - y = y^2(\sin x + \cos x)$.

   b) Solve the differential equation $(3x^2y^3e^y + y^3 + y^2)dx + (x^3y^3e^y - xy)dy = 0$.

4. a) Solve $(8D^2 - 14D + 5)y = 16\sin x$.

   b) Solve $(D^2 + 4D + 4)y = e^{-2x}\sin x$ using method of variation of parameters.

5. a) Evaluate $\int_0^\infty 2^{-9x^2}\, dx$, using gamma function.

   b) Find the power series solution about $x = 2$ of the differential equation

   $$4y'' - 4y' + y = 0,\ y(2) = 0, y'(2) = 1/e.$$

..2..

6. a) Evaluate $\int_0^\infty te^{-3t} \sin t \, dt$, using Laplace transform.

   b) Using Laplace transform method, solve $y''' + 2y'' - y' - 2y = 0, y(0) = y'(0) = 0, y''(0) = 6$.

7. a) Solve $2x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = x^3$.

   b) Evaluate inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$.

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Code No: G-5012/N/AICTE

FACULTY OF ENGINEERING

B.E. II - Semester (AICTE) (Backlog) (New) Examination, February/ March 2025

Subject: Mathematics - II

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each question carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1. a) Are the following vectors linearly independent or not (2,2,0), (3,0,2), (2,-2,2).

   b) Solve $(x^2 - ay)dx = (ax - y^2)dy$

   c) Find the Integrating Factor of the linear differential equation $\frac{dy}{dx} + \frac{x}{1-x^2}y = \frac{ax}{1-x^2}$.

   d) Check whether the functions $\log x$, $\log x^2$ are linearly independent or not.

   e) Evaluate $\int_0^\infty \sqrt{x}\, e^{-x^2}\, dx$ in terms of Gamma functions.

   f) Classify the singular points of $(1-x^2)y'' - 2xy' + 2y = 0$

   g) Find the Inverse Laplace transform of $\frac{1}{(s+1)(s+2)}$.

2. a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -1 \\ 2 & -4 & -4 \end{bmatrix}$. Hence find A^{-1}.

   b) Find the values of a and b for which the equations $x+y+z=3, x+2y+2z=6, x+ay+3z=b$ have (i) no solution (ii) a unique solution (iii) Infinite no. of solutions.

3. a) Show that the one parameter family of curves $y^2 = 4c(c+x)$ are self orthogonal.

   b) Solve $y(2x^2 - xy + 1)dx + (x - y)dy = 0$.

4. a) Solve $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x$.

   b) Solve $x^2y'' - xy' + y = \log x$.

5. a) Prove that $(2n+1)P_n(x) = P'_{n+1}(x) - P'_{n-1}(x)$.

   b) Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

..2..

6. a) Find $L\left[\int_0^t t e^{-t} \sin 4t \, dt\right]$

   b) Solve the differential equation using the Laplace transform. $(D^2+5D+6)x = 5e^t$. Given that $x(0)=2$ and $x'(0)=1$.

7. a) Reduce the quadratic form $3x^2-2y^2-z^2-4xy+8xz+124yz$ to canonical form by orthogonal transformation. Write also nature and index.

   b) It is known that $\frac{1}{x}$ is a solution of the differential equation $x^2y''+4xy'+2y=0$. Find the second linearly independent solution and write the general solution.

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